Fiscal policy: Ricardian Equivalence, the effects of government spending, and debt dynamics

1 The intertemporal government budget constraint

Consider the usual two period model, with constant real interest rate is constant. Assume the government starts in period 1 with some initial net debt inherited from the past, $B_0$. Then the government budget constraint in period 1 is:

$$B_1 = (1 + r)B_0 + G_1 - T_1$$  \hspace{1cm} (1)

Similarly in period 2 it is

$$B_2 = (1 + r)B_1 + G_2 - T_2$$  \hspace{1cm} (2)

As usual, it must be $B_2 = 0$. The government cannot end with positive net debt, because this would mean that some creditor would not get repaid. But it is not optimal for the government to end with negative net debt (i.e., positive net assets). Therefore setting $B_2 = 0$ in (2) and replacing $B_1$ in (1) gives the government intertemporal budget constraint:

$$B_0(1 + r) = T_1 - G_1 + \frac{T_2 - G_2}{1 + r}$$  \hspace{1cm} (3)

$T_i - G_i$ is the primary surplus in period $i$. The primary surplus is taxes less government spending (excluding interest payments). Hence the intertemporal government budget constraint says that: initial net debt (including interest) = present value of primary surpluses.
2 Ricardian Equivalence

Let disposable income in the two periods be

\[ y_1 = \bar{y} - T_1 \]  
\[ y_2 = \bar{y} - T_2 \]  

Consumption of the individual in the two periods is

\[ c_1 = \bar{y} - s - T_1 \]  
\[ c_2 = \bar{y} + s(1 + r) - T_2 \]  

Where \( s \) is savings and \( \bar{y} \) is constant labor income. We assume for simplicity that \( G_1 = G_2 = G \), i.e. government spending is the same in the two periods. We also assume that the initial debt is 0: \( B_0 = 0 \)

The utility function is

\[ U = u(c_1) + \frac{1}{1 + \rho} u(c_2) \]  

Assume for simplicity that \( r = \rho \). Note that in this model government spending does not enter the utility function of the individual directly. Hence, it affects consumption and utility of the individual only through taxation: higher government spending means higher taxation and lower disposable income of the individual. This is the wealth effect of government spending.

For simplicity, we assume that the individual has no initial financial wealth. We consider two cases.

**Case I**

We denote the outcome of this case with a "*". The government sets \( T_1^* = T_2^* = G \). Hence, in each period the budget is balanced, and there is no debt: \( B_1^* = B_2^* = 0 \). From the Euler equation

\[ u'(c_1) = \frac{1 + r}{1 + \rho} u'(c_2) \]  

Since \( r = \rho \), this implies \( c_1^* = c_2^* \), and therefore \( s^* = 0 \).

Note that government savings is also 0: \( s_G^* = T_1 - G_1 = 0 \). Therefore, national savings too is 0: \( s_{TOT}^* = s^* + s_G^* = 0 \). The consumption of the individual in the two periods is

\[ c_1^* = \bar{y} - s^* - T_1^* \]  
\[ = \bar{y} - G \]  

\[ = \bar{y} - G \]
\[ c_2^* = \bar{y} + s^*(1 + r) - T_2^* \]  
\[ = \bar{y} - G \]  

**Case II**

Now assume that the government implements a tax cut. Instead of \( T_1^* = G \), in period 1 the government taxes \( T_1^{**} = G - \Delta T \), where \( \Delta T > 0 \). From the government budget constraint

\[ B_1^{**} = G - T_1^{**} \]  
\[ = \Delta T \]  

Hence, the government runs a deficit in period 1 and issues debt to finance the tax cut. From (2) in period 2 it must be

\[ T_2^{**} = (1 + r)B_1^{**} + G \]  
\[ = (1 + r)\Delta T + G \]  
\[ = T_2^* + (1 + r)\Delta T \]  

i.e., relative to Case I, the government must issue debt \( \Delta T \) in period 1, and taxes in period 2 must increase by \( \Delta T(1 + r) \) to repay that debt.

Now consider the optimal behavior of the individual. She sees taxes decreasing in period 1 by \( \Delta T \). However, she knows that taxes will increase in period 2 by \( (1 + r)\Delta T \). Hence, her human wealth (the present value of disposable incomes) does not change, because

\[ W^* = y_1^* + \frac{y_2^*}{1 + r} \]  
\[ = \bar{y} - T_1^* + \frac{\bar{y} - T_2^*}{1 + r} \]  

and

\[ W^{**} = y_1^{**} + \frac{y_2^{**}}{1 + r} \]  
\[ = \bar{y} - T_1^{**} + \frac{\bar{y} - T_2^{**}}{1 + r} \]  
\[ = \bar{y} - T_1^* + \Delta T + \frac{\bar{y} - T_2^* - (1 + r)\Delta T}{1 + r} \]  
\[ = \bar{y} - T_1^* + \frac{\bar{y} - T_2^*}{1 + r} \]  
\[ = W^* \]
Since her wealth does not change, her optimal consumption also should not change. In fact, the individual consumer could continue consuming the optimal quantities \( c_1^* = c_2^* \). How? She simply saves the tax cuts. Hence, her savings increase by \( \Delta T \):

\[
s^{**} = s^* + \Delta T = \Delta T
\]

Now consider government savings

\[
s_G^{**} = T_1^{**} - G = T_1^* - \Delta T - G = -\Delta T = s_G^* - \Delta T
\]

Government savings falls by \( \Delta T \), but we have seen that private savings increases by \( \Delta T \), relative to Case I. Hence, total savings is unchanged

\[
s_{TOT}^{**} = s^{**} + s_G^{**} = s^* + \Delta T + s_G^* - \Delta T = s^* + s_G^* = s_{TOT}^*
\]

Hence, national savings does not change. Simply, private savings increase to offset the decline in government savings.

What is her disposable income?

\[
y_1^{**} = \overline{y} - s^{**} - T_1^{**} = \overline{y} - \Delta T - (T_1^* - \Delta T) = \overline{y} - G = y_1^*
\]

\[
y_2^{**} = \overline{y} + s^{**}(1 + r) - T_2^{**} = \overline{y} + \Delta T(1 + r) - (T_2^* + \Delta T(1 + r)) = \overline{y} - G = y_2^*
\]

And the disposable income is also unchanged relative to Case I.

Because nothing real changes, the interest rate does not change either. A tax cut has no real effect, because it does not change the present value of government
spending and therefore it does not change the present value of taxation. It merely reallocates taxes from period 1 to period 2. But the individual can undo this by increasing savings, and can continue doing optimal consumption smoothing like before.

The reason is the following: if the government cuts taxes today by 1 dollar without changing spending at any time, the deficit increases by 1 dollar, but taxes will have to increase by $1+r$ dollars next period. Knowing this, the consumer saves all the initial tax cut; next period, she will earn exactly $1+r$ on this extra savings, which she will be able to use to repay the taxes, without altering her consumption path. Thus, private savings increase by exactly as much as government savings decreases; national savings is not affected; hence the capital stock does not change => consumption, the interest rate etc. do not change.

A frequently heard interpretation of Ricardian Equivalent is that "deficits do not matter". This interpretation can be wrong: deficits do not matter if they are caused by a change in the path of taxation, holding constant the present value of government spending. But if the deficit increases because government spending increases, and the present value of government spending also changes, then deficits do matter.

There are several reasons why Ricardian Equivalence can fail:
1) Finite lives. Individuals do not save the whole tax cut because they expect that it will be future generations that will have to repay today’s debt. Hence, national savings falls.
2) Myopia. Individuals do not internalize today that in the future they will have to repay the debt. Hence, national savings falls.
3) Liquidity constraints. Liquidity constrained individuals consume the whole tax cut, because that’s exactly what they would have liked to do in the first place.
4) Distortionary taxation: timing matters

3 Tax smoothing

Taxes so far were lump-sum, i.e. non-distortionary. Distortionary taxation complicates things considerably, because it can have all sorts of effects: intra-temporal and inter-temporal substitution effects. We consider the simplest possible example.

Assume that the utility function is

$$U(c_1, c_2) = \log c_1 + \frac{1}{1+\rho} \log c_2$$ (44)

Income is fixed, and there is a tax on consumption with rate $\tau_t$. Hence:

$$c_1 = (\bar{y} - s)(1 - \tau_1)$$ (45)
\[ c_2 = (\bar{y} + s(1 + r))(1 - \tau_2) \]  

The consumer takes \( \tau_1 \) and \( \tau_2 \) as given. We can rewrite the utility function as

\[ U(c_1, c_2) = \log(\bar{y} - s) + \log(1 - \tau_1) + \frac{1}{1 + \rho} \log(\bar{y} + s(1 + r)) + \frac{1}{1 + \rho} \log(1 - \tau_2) \]  

The first order condition for the consumer is

\[ \frac{1}{\bar{y} - s} = \frac{1 + r}{1 + \rho} \frac{1}{\bar{y} + s(1 + r)} \]  

and assuming as usual \( \rho = r \)

\[ \bar{y} - s = \bar{y} + s(1 + r) \]  

i.e.

\[ s = 0 \]  

Therefore, optimal consumption is

\[ c_1^* = \bar{y}(1 - \tau_1); \quad c_2^* = \bar{y}(1 - \tau_2) \]  

Hence, if the \( \tau_1 < \tau_2, \ c_1 > c_2 \): the consumer prefers to shift consumption when it is taxes less.

Given this, how should a social planner set the two tax rates in order to maximize the consumer’s welfare? The social planner takes as given (51), and therefore her jobs is to

\[
\max_{\tau_1, \tau_2} U = \log c_1^*(\tau_1) + \frac{1}{1 + \rho} \log c_2^*(\tau_2)
\]

s.t.

\[ c_1^* = \bar{y}(1 - \tau_1); \quad c_2^* = \bar{y}(1 - \tau_2) \]

and the intertemporal government budget constraint

\[ \tau_1 \bar{y} + \frac{\tau_2 \bar{y}}{1 + r} = G_1 + \frac{G_2}{1 + r} \]  

Hence, this is equivalent to solving

\[ \max_{\tau_1, \tau_2} H = \log(1 - \tau_1) + \frac{1}{1 + \rho} \log(1 - \tau_2) \]

s.t. (54). The first order condition is

\[ \frac{1}{1 - \tau_1} = \frac{1 + r}{1 + \rho} \frac{1}{1 - \tau_2} \]
i.e.

\[ \tau_1 = \tau_2 \]  

That is, the optimal tax policy is to do tax smoothing: if taxes are distortionary, it is optimal to smooth distortions over time, i.e. in this case to smooth the tax rate perfectly.

The reason is clear: (55) is increasing and concave in \((1 - \tau_1)\) and \((1 - \tau_2)\), i.e. declining and convex in \(\tau_1\) and \(\tau_2\). Hence, to minimize distortions one should smooth the tax rate.

This means that, if \(G_1\) is high and \(G_2\) is low, it is optimal to run a deficit in period 1 and a surplus in period 2, and vice versa if \(G_1\) is low and \(G_2\) is high. This is the tax smoothing motive for the deficit, which is very different from the Keynesian countercyclical argument for the deficit.

4 The effects of government spending

4.1 The neoclassical model

We now consider the effects of a change in government spending in a model with optimizing agents. To see the main effect, it is enough to consider a static model. We need however to assume that leisure enters the utility function, and the real wage is not constant. The production function is

\[ Y = L^\alpha; \quad \alpha < 1 \]  

where \(L\) is labor supply. The real wage is

\[ w = \alpha L^{a-1} \]  

Assume that the utility function is

\[ u(C, 1 - L) = \theta \log C + (1 - \theta) \log (1 - L) \]  

where \(1 - L\) is leisure. Hence the individual maximizes this utility s.t.

\[ C = wL - T \]  

\[ = \alpha L^a - T \]  

Direct substitution gives the f.o.c.

\[ \frac{\theta}{C} = \frac{1 - \theta}{1 - L} \]
i.e.
\[
\frac{\theta}{\alpha L^\alpha - T} = \frac{1 - \theta}{1 - L}
\]  \hfill (64)

The government budget constraint is, trivially,
\[
G = T
\]  \hfill (65)

Hence, to find the effects of an increase in \( G \) on labor supply and consumption, we need to find the effects of a change in \( T \) on \( L \) from (64). Rewrite (64) as
\[
\theta (1 - L) - (1 - \theta) (\alpha L^\alpha - T) = 0
\]  \hfill (66)

Totally differentiating the lhs we must have
\[
-\theta dL - \alpha^2 (1 - \theta) L^{\alpha-1} dL + (1 - \theta) dT = 0
\]  \hfill (67)

i.e.
\[
[\theta + \alpha^2 (1 - \theta) L^{\alpha-1}] dL = (1 - \theta) dT
\]  \hfill (68)

Therefore
\[
\frac{dL}{dT} > 0
\]  \hfill (69)

and, from (63),
\[
\frac{dC}{dT} < 0
\]  \hfill (70)

What is happening? An increase in \( G \) increases \( T \) via the government budget constraint. This reduces the wealth of the individual, and therefore reduces the purchase of the two "goods" in her utility function, consumption and leisure. As leisure falls, the labor supply curve shifts out, and the real wage declines. This is due to the negative wealth effect of government spending via the increase in taxes. Note that output increases, because \( L \) increases, but welfare declines, because consumption and leisure decline.

### 4.2 Keynesian and neo-keyensian models

We cannot enter into the specifics of neo-keynesian models. But the effects of government spending in these models are in some respects the opposite of that in neoclassical model: consumption and the real wage increase. This is due to the fact that not only the labor supply shifts out because of the wealth effect, as in the neoclassical model, but also the labor demand curve shifts out as aggregate demand increases, so the real wage can increase. As the real wage increases, labor income increases, and if there are liquidity constrained individuals their consumption now increases.
5 The dynamics of debt and deficits

Let’s go back to the government debt accumulation equation, rewritten as

\[ B_{t+1} - B_t = G_t - T_t + rB_t \] (71a)

Assume that there is no inflation, so that nominal and real variables are the same. Start from (71a) and divide both sides by \( Y_t \). Let a tilde indicate a variable as a share of GDP: for instance, \( \tilde{b}_t = \frac{B_t}{Y_t} \).

\[
\frac{B_{t+1}}{Y_{t}} - \frac{\tilde{b}_t}{Y_t} = \frac{\tilde{g}_t}{Y_t} - \frac{\tilde{T}_t}{Y_t} + r\frac{\tilde{b}_t}{Y_t}
\] (72)

\[
\frac{B_{t+1}Y_{t+1}}{Y_{t+1}} - \frac{\tilde{b}_t}{Y_t} = \frac{\tilde{g}_t}{Y_t} - \frac{\tilde{T}_t}{Y_t} + r\frac{\tilde{b}_t}{Y_t}
\] (73)

\[
\tilde{b}_{t+1}(1 + \gamma) - \tilde{b}_t = \frac{\tilde{g}_t}{Y_t} - \frac{\tilde{T}_t}{Y_t} + r\frac{\tilde{b}_t}{Y_t}
\] (74)

where \( \gamma \) is the rate of growth of GDP. Now subtract \( \gamma \tilde{b}_t \) from both sides

\[
\tilde{b}_{t+1}(1 + \gamma) - \tilde{b}_t(1 + \gamma) = \frac{\tilde{g}_t}{Y_t} - \frac{\tilde{T}_t}{Y_t} + r\frac{\tilde{b}_t}{Y_t} - \gamma \tilde{b}_t
\] (75)

\[
\tilde{b}_{t+1} - \tilde{b}_t = -\frac{\tilde{s}_t}{1 + \gamma} + \frac{r - \gamma}{1 + \gamma} \tilde{b}_t
\] (76)

\[
= \frac{\tilde{d}_t}{1 + \gamma} + \frac{r - \gamma}{1 + \gamma} \tilde{b}_t
\] (77)

where \( \tilde{d}_t \) (\( \tilde{s}_t \)) is the primary deficit (surplus) as share of GDP.

We say that fiscal policy is sustainable if the debt/GDP ratio converges to some constant, assuming that the primary deficit or surplus is constant. Rewrite (76) as

\[
\tilde{b}_{t+1} = -\frac{\tilde{s}_t}{1 + \gamma} + \frac{1 + r}{1 + \gamma} \tilde{b}_t
\] (78)

The coefficient of \( \tilde{b}_t \) is less than 1 if \( \gamma > r \); in this case, debt converges to a constant for any constant negative primary surplus \( \tilde{s} \) (if \( \tilde{s} \) is positive, net debt converges to a negative number).

Consider the more interesting case of \( r > \gamma \). Then given a constant surplus \( \tilde{s} \), debt explodes if the primary surplus is smaller than \( b_0(r - \gamma) \), where \( b_0 \) is the initial level of debt, and it implodes in the opposite case.

Thus, given a positive initial level of debt, it gives us the primary surplus needed to keep it stable. That is we need a primary surplus such that

\[
\tilde{s} = b_0(r - \gamma)
\] (79)
Note that the **total** surplus must be negative, i.e. one has a total deficit

\[ \tilde{s} - r\tilde{b}_0 = -\gamma\tilde{b}_0 \quad (80) \]

Note that, given \( \tilde{b}_0 \), to guarantee sustainability one needs a higher primary surplus \( \tilde{s} \), the higher is \( r \), and the lower is \( \gamma \). For instance, to guarantee that a country does not exceed the Maastricht Treaty limit of 60 percent for the debt-to-GDP ratio \( \tilde{b}' = .60 \), if \( r = .03 \) and \( \gamma = .02 \), one needs a primary surplus of at least .006, while the total deficit can be .012 of GDP \((=.006 - .03*.60)\).

Note the distinction between the notions of **solvency** and **sustainability**. Solvency: when the PDV of current and future primary surpluses is enough to repay the initial debt. Sustainability: when the debt/GDP ratio converges to some constant, assuming that the primary deficit or surplus is constant.